Negative and fractional powers

mc-indices2-2009-1

In many calculations you will need to use negative and fractional powers. These are explained on this leaflet.

Negative powers

Negative powers are interpreted as follows:

$$a^{-m} = \frac{1}{a^m} \qquad \text{or equivalently} \qquad a^m = \frac{1}{a^{-m}}$$

Examples

$$3^{-2} = \frac{1}{3^2}, \qquad \frac{1}{5^{-2}} = 5^2, \qquad x^{-1} = \frac{1}{x^1} = \frac{1}{x}, \qquad x^{-2} = \frac{1}{x^2}, \qquad 2^{-5} = \frac{1}{2^5}$$

Exercises

1. Write the following using only positive powers:

(a) $\dfrac{1}{x^{-6}}$, (b) x^{-12}, (c) t^{-3}, (d) $\dfrac{1}{4^{-3}}$, (e) 5^{-17}.

2. Without using a calculator evaluate (a) 2^{-3}, (b) 3^{-2}, (c) $\dfrac{1}{4^{-2}}$, (d) $\dfrac{1}{2^{-5}}$, (e) $\dfrac{1}{4^{-3}}$.

Fractional powers

To understand fractional powers you first need to have an understanding of roots, and in particular square roots and cube roots.

When a number is raised to a fractional power this is interpreted as follows:

$$a^{1/n} = \sqrt[n]{a}$$

So,

$$\begin{array}{ll} a^{1/2} & \text{is a square root of } a \\ a^{1/3} & \text{is the cube root of } a \\ a^{1/4} & \text{is a fourth root of } a \end{array}$$

Examples

$$3^{1/2} = \sqrt[2]{3}, \qquad 27^{1/3} = \sqrt[3]{27} \text{ or } 3, \qquad 32^{1/5} = \sqrt[5]{32} = 2,$$

$$64^{1/3} = \sqrt[3]{64} = 4, \qquad 81^{1/4} = \sqrt[4]{81} = 3$$

Fractional powers are useful when we need to calculate roots using a scientific calculator. For example to find $\sqrt[7]{38}$ we rewrite this as $38^{1/7}$ which can be evaluated using a scientific calculator. You may need to check your calculator manual to find the precise way of doing this, probably with the buttons x^y or $x^{1/y}$.

Check that you are using your calculator correctly by confirming that

$$38^{1/7} = 1.6814 \quad (4\,\mathrm{dp})$$

More generally we can write:

$$\boxed{a^{m/n} = \sqrt[n]{a^m} \text{ or equivalently } \left(\sqrt[n]{a}\right)^m}$$

Examples

$$8^{2/3} = (\sqrt[3]{8})^2 = 2^2 = 4, \qquad \text{and} \qquad 32^{3/5} = (\sqrt[5]{32})^3 = 2^3 = 8$$

Alternatively,

$$8^{2/3} = \sqrt[3]{8^2} = \sqrt[3]{64} = 4, \qquad \text{and} \qquad 32^{3/5} = \sqrt[5]{32^3} = \sqrt[5]{32768} = 8$$

Exercises

3. Use a calculator to find, correct to 4 decimal places, a) $\sqrt[5]{96}$, b) $\sqrt[4]{32}$.
4. Without using a calculator, evaluate a) $4^{3/2}$, b) $27^{2/3}$.
5. Use the rule $\dfrac{a^n}{a^m} = a^{n-m}$ with $n = 0$ to prove that $a^{-m} = \dfrac{1}{a^m}$.
6. Each of the following expressions can be written as a^n. Determine n in each case:

(a) $\dfrac{1}{a^5}$ (b) $\sqrt{a} \times \dfrac{1}{a^2}$, (c) 1 (d) $\dfrac{1}{\sqrt{a}}$.

Answers

1. (a) x^6, (b) $\dfrac{1}{x^{12}}$, (c) $\dfrac{1}{t^3}$, (d) 4^3, (e) $\dfrac{1}{5^{17}}$.
2. (a) $2^{-3} = \dfrac{1}{2^3} = \dfrac{1}{8}$, (b) $\dfrac{1}{9}$, (c) 16, (d) 32, (e) 64.
3. a) 2.4915, b) 2.3784. 4. a) $4^{3/2} = 8$, b) $27^{2/3} = 9$.
6. (a) -5 (b) $-\frac{3}{2}$ (c) 0 (d) $-\frac{1}{2}$.

